

UNSTEADY-STATE AND STEADY-STATE
CONTINUOUS THICKENING

LARS-GÖRAN EKLUND

CI, GB

AVHANDLING FÖR TEKNISK DOKTORSEXAMEN

1976



DEPARTMENT
OF
CHEMICAL ENGINEERING



AVDELNINGEN FÖR KEMISK APPARATTEKNIK

LUND INSTITUTE OF TECHNOLOGY
LUND - SWEDEN



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The dynamic behaviour of a continuous thickener at stepwise changes of solids load has been studied at thickening of unflocculated barium sulphate. Different models for the dynamics have been formulated and discussed. The simulated responses of the models have been compared with the experimentally found results.

Further the working conditions in a pilot thickener at steady-state and optimal load were investigated at thickening of unflocculated calcium carbonate. The theory of a capacity restricting critical solids flux was not found to be valid. The deviations were explained by non-ideal thickener behaviour caused by the incoming feed.

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UNSTEADY-STATE AND STEADY-STATE CONTINUOUS THICKENING

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This thesis consists of a literature survey of unsteady-state thickening, a discussion of models for the dynamic behaviour of an optimally loaded thickener and summaries of the following papers:

I Eklund, L.-G. and Jernqvist, Å.

Experimental Study of the Dynamics of a Vertical Continuous Thickener - I.

Chem. Engng Sci. 1975 30 597.

II Eklund, L.-G.

A Critical Investigation of the Working Conditions in a Continuous Pilot Thickener at Optimal Load.

Dept. of Chem. Eng., Lund Institute of Technology, Report no 75-F-4, 1975. (In press in slightly revised form in Chem. Engng Sci.)

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INTRODUCTION

Processes for separating solid substances from liquids are widely encountered nowadays. With increasing demands from environmental authorities for low impurity quantities in effluents from industrial and municipal wastewater treatment systems a great need for efficiently working separation processes has arisen.

Since long, gravity separation has been used for separating solids from liquids. Even with soluble and colloidal substances, it is possible to apply gravity settling if the substance is first converted to an insoluble and settleable form. This is done in the activated sludge process, where colloidal and soluble organic material is converted by microorganisms to settleable cell substance, as well as in the tertiary wastewater treatment process, where colloidal and soluble substances are precipitated by adding a flocculating agent.

In gravity settling processes, two requirements are to be fulfilled. Firstly, the effluent liquid must have a negligible amount of solids. Secondly, the concentrated slurry must have a sufficiently high solids concentration. The primary function of a solid liquid gravity separator is mostly to produce a clear liquid effluent while the thickening function of such a unit can not be underestimated. The process economy in subsequent sludge handling, e.g. sludge disposal and filtration, will be greatly affected by the sludge volume from the settler.

In this thesis, the efforts have been directed towards the thickening process in solid-liquid gravity separators.

Part I of the thesis is concerned with the dynamic behaviour of a continuous thickener. The following reasons elucidate the importance of a correct understanding of the dynamic behaviour of a thickener:

- a. In most systems feed flow rate and feed flow quality change with time. The changes can be deterministic as well as stochastic.
- b. In order to obtain a good process control and a proper choice of control strategy, it is necessary to know the dynamic behaviour of the process as well as the behaviour of the controllers.

c. In the design of a thickener, it should be possible to base the designing on a dynamic model and on the deterministic flow behaviour. Today, the designing is usually based on an average feed flow parameter.

In Part I, experimental results from step changes of feed flow rate to a pilot thickener and a model for the dynamic behaviour are presented. The following discussion will include different theoretically conceivable models, which have given poor agreement with the experimental results, and therefore will not be found in Part I.

While developing and refining the model presented in Part I, it was found that the basic theories concerning the behaviour and capacity-restrictions of a continuous thickener were not fully valid in the pilot thickener used. Indications were found, that the capacity was not restricted by the minimum of the total solids flux and that the underflow concentration at optimal load was not solely a function of solids load, independent of feed concentration. This discovery was contrary to theories accepted since the early sixties [1, 2, 3].

To investigate if these indications were correct and, if so, to what extent the deviations from accepted theories occurred, a thorough analysis of the working conditions were made in the pilot thickener at different steady states at optimal load.

In Part II, the results from this investigation are presented. The results obtained confirm the previously found indications.

UNSTEADY-STATE CONTINUOUS THICKENING

The dynamic behaviour of thickeners has attracted little interest in literature and what has been published has appeared during the last five years. The most complete analysis of thickening dynamics up till now, has been made by Tracy [4], who formulated a dynamic model and compared its simulated responses with experimental data obtained in a laboratory thickener. Prior to Tracy, dynamic models of continuous thickeners had been formulated by Alkema [5] and by Bryant and Wilcox [6]. As neither of them has supported their models with experimental data and as Tracy's model is based on the essential elements developed by Alkema and by Bryant and Wilcox, only his model will be discussed below.

Tracy's model

Tracy's model is based on a fundamental mass balance and on the solids flux theories for an ideal slurry in a continuous thickener. The settling theories for an ideal slurry have been presented by Kynch [7], and the thickening process of such slurries in continuous thickeners has been presented by Yoshioka et al. [1]. For an ideal slurry, there exists a well defined relationship between solids flux by settling, S , and solids concentration. In a continuous thickener, the downward solids flux will consist of two components. One is the above mentioned flux by settling, and the other is the induced flux by the downward movement of the slurry mass. This is shown below, where the bellshaped curve is the flux by settling and the straight line is the induced flux at a specific slurry underflow velocity, L/A .

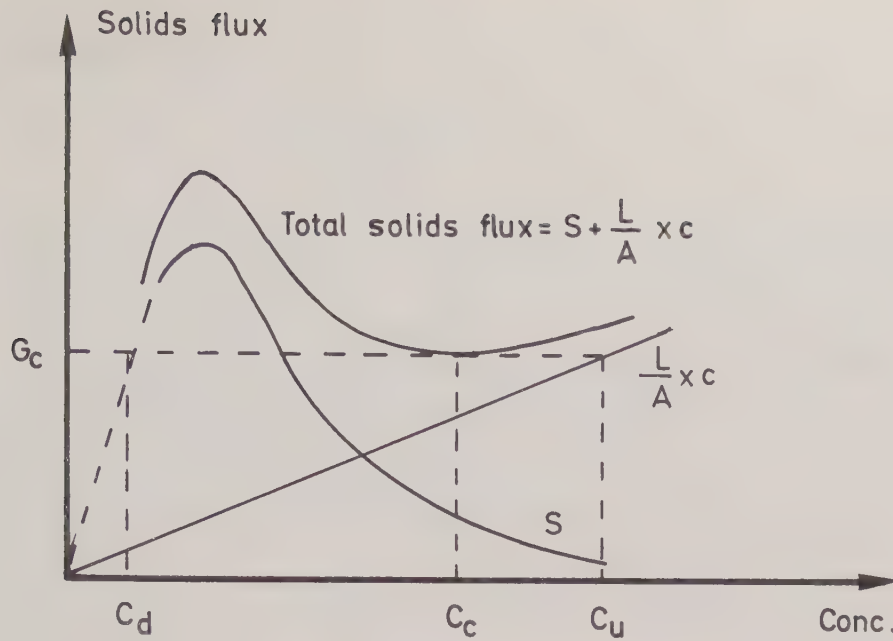


Figure 1. Induced solids flux, solids flux curve by settling and total solids flux curve.

The capacity-limiting solids flux at steady-state is obtained at the minimum point of the total solids flux curve, G_c .

Regard the thickener in Fig. 2 below.

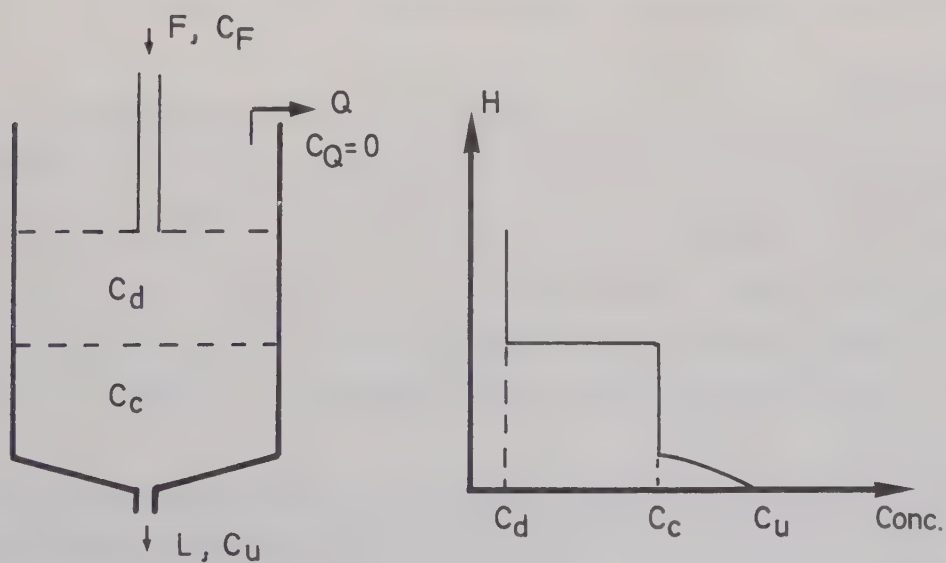


Figure 2. Operating conditions in an ideal thickening system at optimal load.

With a solids load

$$G = \frac{F}{A} \times c_F = G_C \quad (1)$$

the thickener is optimally loaded and the input solids exactly match the solids flux capacity of the thickener. There will be a dilute zone with the concentration c_d , and a critical zone with the concentration c_c . The transition between the two zones can be located anywhere in the cylindrical portion below the feed well.

As the solids load to the thickener

$$G = \frac{F}{A} \times c_F < G_C \quad (2)$$

the capacity of the thickener is greater than its load. A dilute concentration will prevail in the thickener and there will be no critical zone.

With a solids load

$$G = \frac{F}{A} \times c_F > G_C \quad (3)$$

the thickener will be overloaded and, as a consequence, the applied solids load cannot pass from the feed well to the underflow. The solids flux, which is not allowed to pass to the underflow, is $G - G_C$.

For a thickener, working under optimal load, in which the load is changed to above G_C , an accumulation of solids will occur, as all solids cannot pass to the underflow. A zone with the critical concentration will develop at the bottom of the thickener and grow upward, thereby reducing the height of the dilute zone, reach the feed well and eventually give a deteriorated effluent. At this stage, the thickener has reached a steady-state under overloaded conditions.

The opposite phenomenon, with a gradually decreasing critical zone, will occur if the applied solids load is decreased from above G_C to below G_C .

As can be seen from Fig. 1, it is possible to change the total solids flux curve by changing the underflow velocity L/A . An increase of L/A will increase G_c and vice versa. Thus, an increase of the applied solids load above the prevailing G_c can be matched by increasing L/A . As can be seen in Fig. 1, this will result in a decreased underflow concentration.

In order to find the transient conditions in the thickener, Tracy formulated a solids mass balance over a layer of height Δz with uniform concentration and positioned below the feed well. See fig. 3!

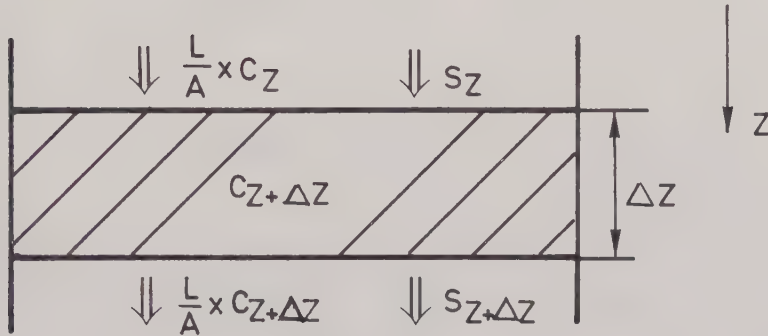


Figure 3. Solids flux conditions for a height layer in a thickener.

A solids mass balance gives:

$$A \times \Delta z \times \frac{dc_{z+\Delta z}}{dt} = A \times \left(\frac{L}{A} \times c_z + S_z - \frac{L}{A} \times c_{z+\Delta z} - S_{z+\Delta z} \right) \quad (4)$$

$$\frac{dc_{z+\Delta z}}{dt} = - \frac{L}{A} \times \frac{\Delta c_z}{\Delta z} - \frac{\Delta S_z}{\Delta z} \quad (5)$$

With the terms expanded in a Taylor series and Δz assumed infinitesimally small Eq. (5) can be written as:

$$\frac{\partial c}{\partial t} = - \left(\frac{L}{A} + \frac{dS}{dc} \right) \times \frac{\partial c}{\partial z} \quad (6)$$

Eq. (6) was originally formulated by Shannon et al. [2]. The derivative dS/dc in Eq. (6) must for a non-ideal slurry, for which by definition S is not a well defined function of c , be replaced by the partial derivative $\partial S/\partial c$.

Tracy modeled the thickener according to Fig. 4 below.

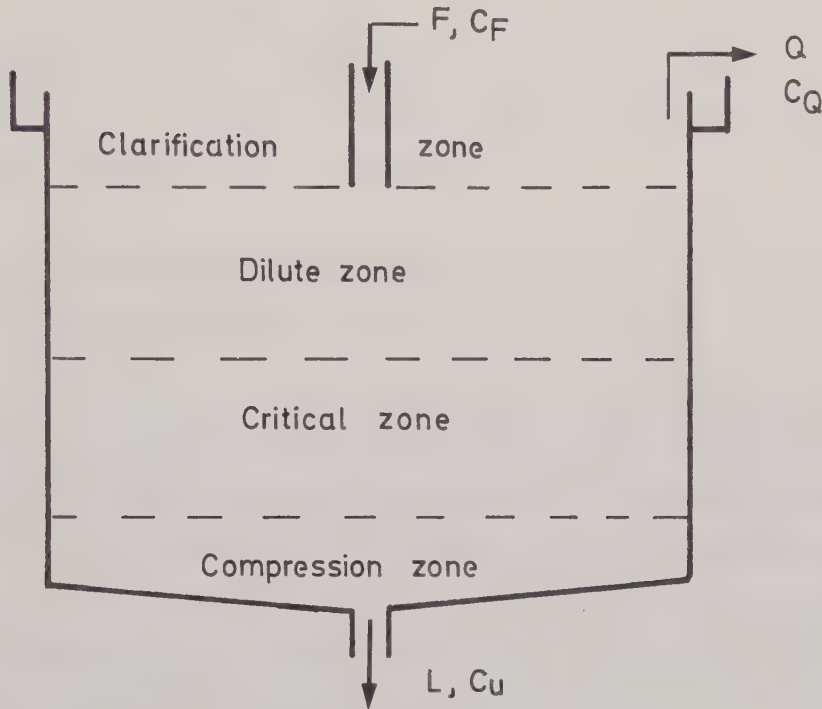


Figure 4. Tracy's model of a thickener.

The bottom layer is termed the compression zone, and is assumed to exist with constant height during each simulated transient. In this zone, the settling flux by gravity is zero, and it is regarded as a completely mixed vessel with the underflow concentration c_u . By setting $S_{z+\Delta z}$ in Fig. 3 equal to zero and with $c_u = c_{z+\Delta z}$, it is obtained:

$$\frac{dc_u}{dt} = \frac{L}{A} \times \frac{c_z}{\Delta z} + \frac{S_z}{\Delta z} - \frac{L}{A} \times \frac{c_u}{\Delta z} \quad (7)$$

Above the compression zone, the critical zone may exist. This zone is present either when the solids load is greater than or equal to G_c , or when, after a period of overload, the conditions have been changed to underloaded and the solids in the critical zone have not yet passed to

the compression zone.

The critical zone was modeled as a series of layers with constant concentrations. The top layer of the critical zone could be allowed to grow upward at overload or fall towards the bottom at underload. By changing the operating conditions, i.e. the solids load or the underflow, with the thickener still overloaded, a new critical layer will develop on top of the previous one. Each developed layer will keep its identity with constant concentration and move downward through the critical zone. The mass balance according to Eq. (5) will be fulfilled by letting for each layer $dc_z/dt = 0$ and by adjusting the height, Δz , of the layer.

With this technique, it is possible to follow the movement of the critical zone by adding the heights of all layers within it.

The great drawback of this model is its inapplicability when imposing an operating change which creates a critical layer with a concentration higher than the concentration of the layer below. With such a concentration profile, density currents should arise.

The dilute zone, positioned above the critical one, was modeled as three completely mixed vessels in series. Its height was variable and depending on the height of the critical zone. The concentrations are obtained by Eq. (5). For the top layer in the dilute zone Eq. (5) is modified to:

$$\frac{dc}{dt} = \frac{F \times c_F}{A \times \Delta z} - \frac{L}{A} \times \frac{c_{z+\Delta z}}{\Delta z} - \frac{S_{z+\Delta z}}{\Delta z} \quad (8)$$

As Tracy has pointed out, it should be possible to neglect the dilute zone and assume that the solids pass directly by density currents from the feed well to the top of the critical zone. This version of the model has been simulated by Tracy and Keinath [8].

After a prolonged overload, the critical zone shall have grown from the bottom to the feed inlet. At this stage a flux:

$$G_{c1} = \frac{F}{A} \times c_F - G_c \quad (9)$$

will advance upward through the clarification zone. Tracy assumed the concentration in this zone to be equal to the critical concentration prevailing below the feed inlet.

With the solids flux curve by settling of the ferric hydroxide slurry used, Tracy could simulate step changes of the operating conditions. The responses were compared with data from experimentally performed step changes. Tracy found that the model, at these step changes, predicted the underflow concentration response under both underloaded and overloaded operational conditions. However, in predicting the height of the critical zone the accuracy was found to be limited.

Experimental work

Tracy's model was based on a thickener in which solids load and underflow rate could be independently varied.

In the work presented in Part I, step changes of solids load have been made in a thickener working at optimal load. This optimal load was achieved by regulating the underflow pump so that the concentration discontinuity between the dilute and the critical zones was kept on a constant level. Thus the underflow rate was a function of the solids load with a transient behaviour at load changes. This fact must be included in a dynamic model of the thickener. Therefore, a different approach to the dynamic modeling than that used by Tracy, is necessary.

The continuous thickening experiments were performed with the equipment shown below.

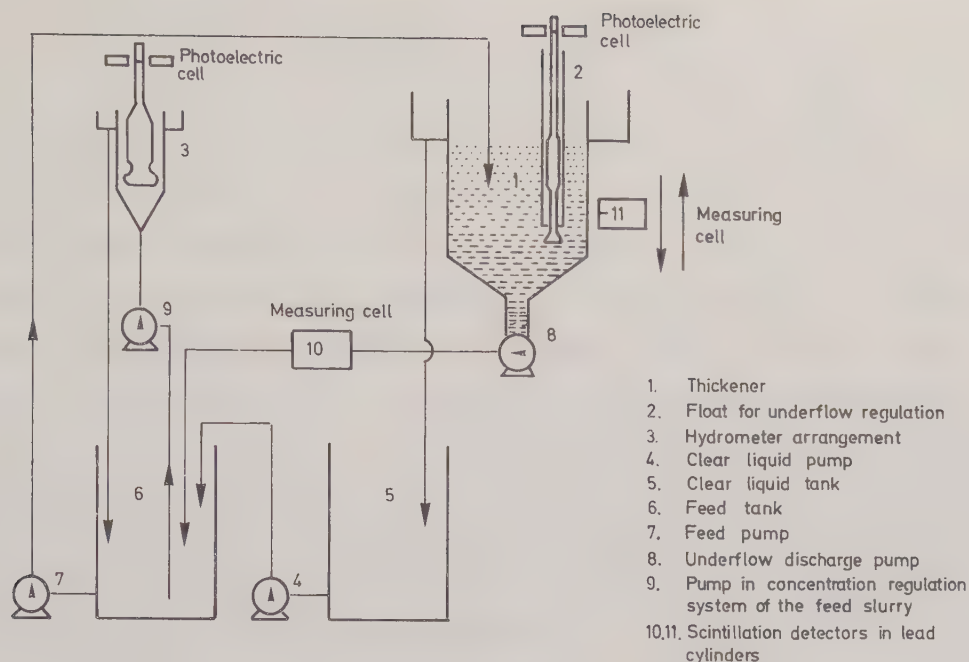


Figure 5. Experimental set up used at unsteady-state thickening.

The diameter of the thickener is 0.3 m. The c_c/c_d -concentration discontinuity was held constantly at 0.43 m above the cone during all experimental runs. Above the discontinuity, the dilute zone reached to the feed inlet with the height 0.26 m.

A slurry of unflocculated barium sulphate in water was chosen for the experiments. With trace amounts of the radioactive isotope ^{131}Ba , it was possible to apply a radioactive measuring technique and continuously measure the underflow concentration. Also, the concentration in the thickener was measured by a mobile measuring cell at the wall of the thickener. The results obtained from this cell, however, proved to be of limited accuracy.

The solids load applied to the thickener was varied stepwise in the interval $4.4 - 9.0 \text{ kg/m}^2 \text{ h}$ with a constant solids concentration of 21.2 kg/m^3 in the feed.

Models for the dynamic behaviour

In formulating a dynamic model for continuous thickening, the solids flux curve by settling must be known. As mentioned above, there exists for an ideal slurry a well defined relationship between solids flux by settling and solids concentration. As the barium sulphate slurry was assumed to be ideal, it should have been possible to find the solids flux curve under batch conditions. However, the solids flux curves obtained from batch experiments, were found to differ, indicating that the slurry was not perfectly ideal. Therefore, the solids flux curve by settling, valid in the thickener, was derived from the continuous thickening data. Analytical expressions, giving the critical solids flux by settling and the critical concentration, were derived as functions of a well defined relationship between solids load and underflow concentration at optimal load. These expressions were used in the dynamic models discussed below.

A conceivable dynamic model of a continuous thickener at optimal load is formulated below. The model is based on ideal slurry behaviour. The thickener is assumed to have a well defined relationship between solids load and underflow concentration and there exists no radial concentration variation within the thickener.

The different zones in the thickener are shown below.

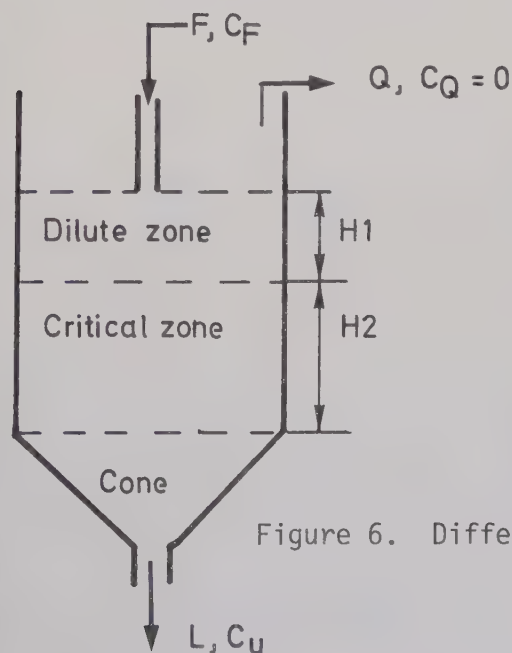


Figure 6. Different zones in thickener.

H1 and H2 are constant, thus keeping the concentration discontinuity between the two zones on a constant level.

The dilute zone is modeled as a perfectly stirred vessel with a uniform concentration. Thus with a mass balance over this zone one obtains:

$$\frac{dc_d}{dt} = \left\{ \frac{F}{A} \times c_F - \frac{L}{A} \times c_d - S_d \right\} \times \frac{1}{HT} \quad (10)$$

The S_d/c_d -relationship is achieved from continuous thickening data from the dilute zone at steady-states. As mentioned above, the underflow rate, L , will show a transient behaviour at load changes. A model for this behaviour must be incorporated in the dynamic model of the thickener. Such a model will be discussed and formulated further down in the text.

In the critical zone, ideal thickening behaviour is assumed to prevail at unsteady-states. This means that the solids are instantaneously thickened to the concentration prevailing at the top of the critical zone when passing from the dilute zone. After that, the solids move continuously without any mixing in a variable concentration field towards the cone. The slurry is regarded as ideal with a well defined solids flux curve by settling. Hence, solids compression forces are assumed not to exist in the critical zone. A proper model for this zone is obtained by dividing it into a series of layers with uniform concentrations. See below!

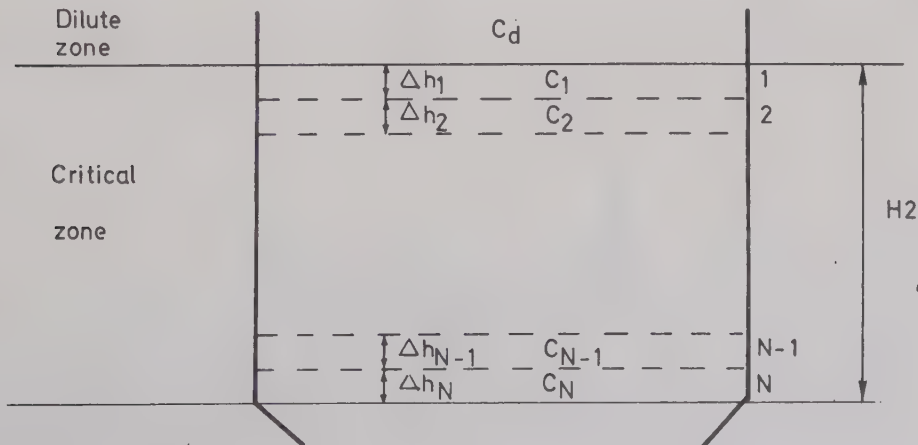


Figure 7. A model for the critical zone.

For the top layer with $i=1$ a mass balance gives:

$$\frac{dc_1}{dt} = \left\{ \frac{L}{A} \times c_d + S_d - \frac{L}{A} \times c_1 - S_1 \right\} \times \frac{1}{\Delta h_1} \quad (11)$$

and for the lower layers with $2 \leq i \leq N$:

$$\frac{dc_i}{dt} = \left\{ \frac{L}{A} \times c_{i-1} + S_{i-1} - \frac{L}{A} \times c_i - S_i \right\} \times \frac{1}{\Delta h_i} \quad (12)$$

The height of the critical zone is constant:

$$\sum \Delta h_i = H_2 \quad (13)$$

The solids flux by settling, S_i , in Eqs. (11) and (12) is equal to the critical solids flux by settling, derived from continuous steady-state thickening data.

The applicability of the model above is restricted in two ways. Firstly, the concept of a limiting solids flux must not be violated so that a layer concentration passes through the critical concentration at the specific underflow rate. Secondly, the concentration in one layer must not attain a higher value than the concentration of a layer below. Such an inverted concentration profile would give rise to density currents with a resulting equalizing of the concentration within the critical zone. These restrictions will be discussed in connection with the modeling of the underflow behaviour at unsteady-state.

At this initial stage of formulating a dynamic model, the existence of the cone is neglected according to the figure below.

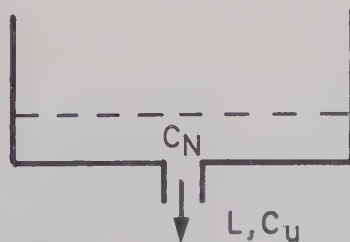


Figure 8. A model for the bottom of the thickener.

The underflow concentration is found from:

$$c_u = c_N + S_N/L \quad (14)$$

While modeling the transient behaviour of the underflow let us first regard the conditions at steady-state and optimal load. Under these conditions, the underflow is regulated by the movement of the c_d/c_c -discontinuity, according to:

$$\frac{L}{A} = - \frac{dS_c}{dc_c} \quad (15)$$

$$\frac{L}{A} = - \frac{S_d - S_c}{c_d - c_b} \quad (16)$$

One plausible model of the underflow rate is found by regarding the uppermost layer of the critical zone which with Eq. (15) gives:

$$\frac{L}{A} = - \left. \frac{dS_c}{dc_c} \right|_{c_c = c_1} \quad (17)$$

The prohibition of an inverted concentration profile limits the applicability of the model put forward above. It is realized that such inverted profiles will develop while simulating a decreased solids load and therefore the model is restricted to transients caused by an increased solids load.

One possibility to avoid inverted concentration profiles is to model the critical zone as a single layer of uniform concentration. According to Eq. (17), the underflow rate will be a function of this concentration. Thus, the fundamental assumption, that the underflow rate will be dependent on specific conditions at the c_d/c_c -discontinuity, will not be valid.

At an increased solids load, an inverted concentration profile will not develop and so the critical zone can be modeled as a series of uniform concentration layers. In this case, as seen by Eqs. (11) and (17), the transient of the underflow rate will be a function of the layer height, Δh_1 . Data for the underflow rate at unsteady-state are therefore necessary to evaluate a proper value of Δh_1 . In the experimental work reported in Part I, such data were not obtained.

A different approach to modeling the underflow rate is achieved by regarding an infinitesimally thin layer at the top of the critical zone. This layer will, from now, be termed the critical layer. As the height of the critical layer is zero, it will be in solids flux balance with the dilute zone. This is realized by regarding Eq. (11). Thus the concentration in the critical layer will be equal to the critical concentration at steady-state with the existing dilute concentration. By Eqs. (15) and (16) which give:

$$\frac{dS_c}{dc_c} = \frac{S_d - S_c}{c_d - c_c} \quad (18)$$

the critical concentration can be calculated as a function of the dilute concentration. The underflow rate is then calculated from Eq. (15) or (16). By regulating the underflow rate from the concentration of a hypothetical layer of zero thickness, the response obtained at load changes is faster than the corresponding response with a top layer of finite thickness in the critical zone governing the underflow rate.

The model discussed above, with a critical layer regulating the underflow rate, has been simulated at step changes of the solids load.

The initial values of c_c , L/A , c_d , and c_u were found from:

$$\frac{F}{A} \times c_F = - \frac{dS_c}{dc_c} \times c_c + S_c \quad (19)$$

$$\frac{L}{A} = - \frac{dS_c}{dc_c} \quad (20)$$

$$\frac{F}{A} \times c_F = \frac{L}{A} \times c_d + S_d \quad (21)$$

$$F \times c_F = L \times c_u \quad (22)$$

The experimentally found expressions:

$$S_c = S(c_c) \quad (23)$$

and

$$S_d = S_d(c_d) \quad (24)$$

are given in Part I.

The thickener response at a step change of the solids load from 4.29 to 8.52 kg/m² h is shown in Fig. 9. The experimentally obtained underflow concentration response and the simulated underflow concentration responses with the critical zone divided into 1, 2, 4, and 8 equally high zones are plotted. The concentration in the critical layer, c_c , is also plotted in the figure.

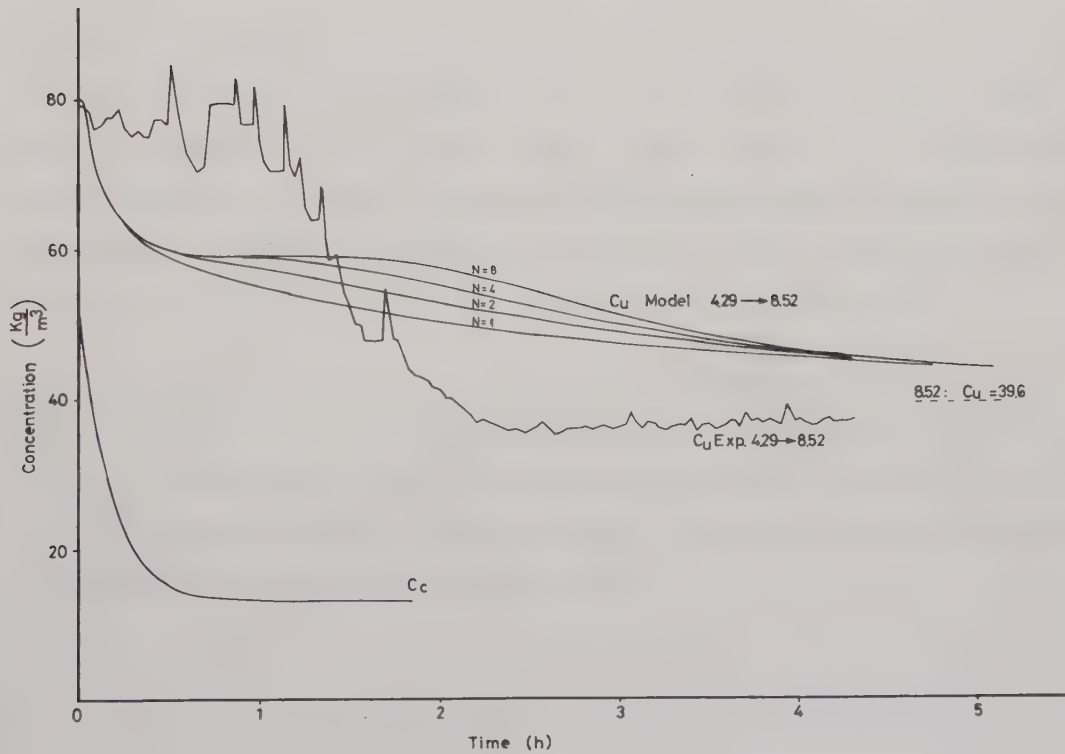


Figure 9. Simulated step responses of c_u and c_c and the experimental step response of c_u to solids load change from 4.29 to 8.52 kg/m² h.

As can be seen, the step response of the critical layer is independent of the number of layers in the critical zone.

The step responses of the simulated underflow concentrations are initially fast with decreasing concentrations. This is due to the fast response of the underflow rate, which effects the underflow concentration according to Eq. (14). When the fast decrease of the underflow rate has ceased, the change of the underflow concentration with time will decrease and the responses for the models with 1, 2, 4, and 8 layers in the critical zone will differ.

The agreement between the experimental and simulated underflow responses is poor. It is not possible to find any indication in the experimental response of the tendency of the simulated response, at a high number of layers, to form a constant concentration period after the initial fast decrease.

Regarding the restriction, that the concentration in a layer of the critical zone must not pass through the critical concentration, it is realized that the restriction is not violated with this model. The response of the critical layer, shown in Fig. 9, is faster than the responses of the different layer concentrations. Consequently, it gives a critical concentration which is lower than the concentrations in the critical zone during the transient period.

Also a solids load decrease from 8.52 to 4.29 kg/m² h has been simulated. For this step change there are no experimental data. Due to the restriction against inverted concentration profiles, the model was simulated only with one layer in the critical zone. The result of the simulation is shown in Fig. 10.

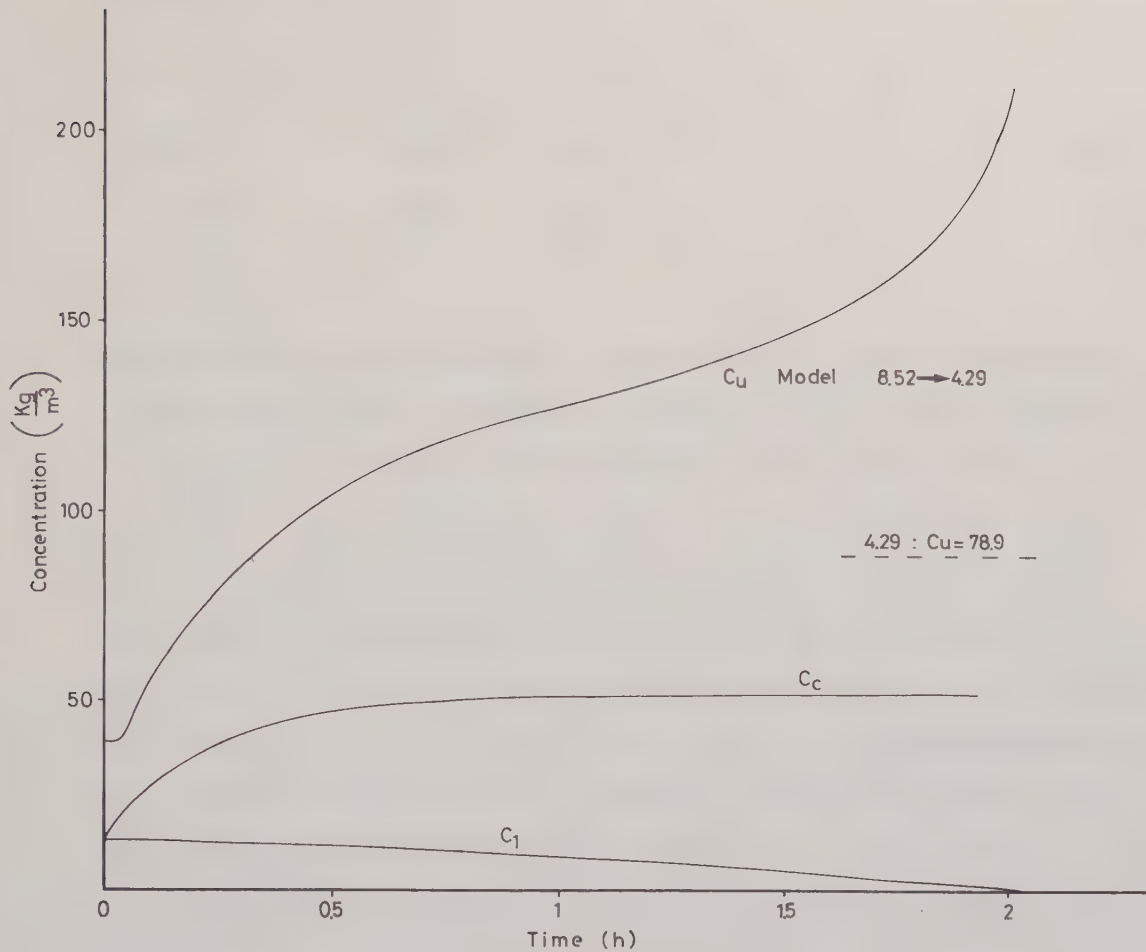


Figure 10. Simulated step responses of c_u , c_c and c_l to solids load change from 8.52 to 4.29 kg/m² h.

In Fig. 10, the responses of the underflow concentration, the critical concentration and the concentration in the critical zone, c_l , are plotted. It is obvious that the model does not give a correct response. The value of c_u grows toward infinity, while c_l decreases below zero. This behaviour can be explained by regarding the solids flux conditions in the critical zone at steady-states with the solids loads 4.29 and 8.52 kg/m²h in Table 1 below.

Table 1

Solids load kg/m ² h	L/A m/h	c _c kg/m ³	c _c × L/A kg/m ² h	S _c kg/m ² h
4.29	0.055	52.4	2.85	1.45
8.52	0.217	13.2	2.84	5.69

When the solids load changes stepwise to 4.29 kg/m² h, the dilute concentration and the underflow rate will give a faster response than the concentration in the critical zone. Hence, the solids flux 4.29 kg/m² h from the dilute zone to the critical one is attained relatively fast. With a solids flux by settling from the critical zone which still is approximately 5.69 kg/m² h, there will be a net solids outflow from the critical zone. As the derived critical solids flux by settling grows with decreasing solids concentration, the concentration c₁ in the critical zone will decrease continuously, while the underflow concentration increases according to Eq. (14). The critical solids flux curve by settling is only defined in a limited concentration interval which does not overlap the decreasing concentration in the critical zone. With a correct form of the solids flux curve by settling, according to Fig. 1, a different and more correct response would have been obtained.

Thus, it should be possible to modify the model in order to get a transient towards correct steady state values. However, this will not be done here. Instead, another form of a dynamic model which gives a better agreement with experimental data will be formulated.

The dynamic model in Part I

At steady-state thickening the relationship:

$$\frac{L}{A} \times c_u = S_c + \frac{L}{A} \times c_c \quad (25)$$

between the critical concentration and the underflow concentration is valid. As, according to Eq. (15), the underflow rate is a function of the critical concentration, the underflow concentration can be regarded as a function of the critical concentration. It is possible to derive a relationship between these two variables from steady-state

continuous thickening data. The derivation is based on the concept of a well defined capacity-restricting critical solids flux by settling.

Thus, from steady-state data it is obtained a relationship:

$$c_u = f(c_c) \quad (26)$$

In the dynamic model presented in Part I, Eq. (26) is incorporated. The critical zone is modeled as a completely stirred vessel with the concentration c_a . Neglecting the existence of the cone and using Eq. (26) the underflow concentration is obtained from:

$$c_u = f(c_a) \quad (27)$$

The underflow rate is, as in the previous model, regulated by a critical layer of zero thickness, according to Eq. (15). The dilute concentration is obtained from Eq. (10) and the concentration in the critical layer from Eq. (18). With a mass balance over the critical zone c_a can be calculated from:

$$\frac{dc_a}{dt} = \left\{ \frac{L}{A} \times c_d + S_d - \frac{L}{A} \times c_u \right\} \times \frac{1}{H^2} \quad (28)$$

The simulated step responses at solids load changes from 4.29 to 8.52 and from 8.52 to 4.29 $\text{kg/m}^2 \text{ h}$ is shown in Fig. 11. Also, the experimental step response of c_u at the solids load change from 4.29 to 8.52 $\text{kg/m}^2 \text{ h}$ is shown.

Regarding the solids flux conditions the relationship

$$c_u = c_a + S_a / \frac{L}{A} \quad (29)$$

should give the underflow concentration. Eq. (29) is analogous to Eq. (14). Eq. (27) is derived from steady-state thickening data for which Eq. (25) is valid. At steady-state $c_c = c_a$. If Eq. (27) shall give an underflow concentration according to Eq. (29) so the critical layer concentration c_c must equal c_a . This is not the case and therefore Eq. (27) give a response which does not satisfy Eq. (29).

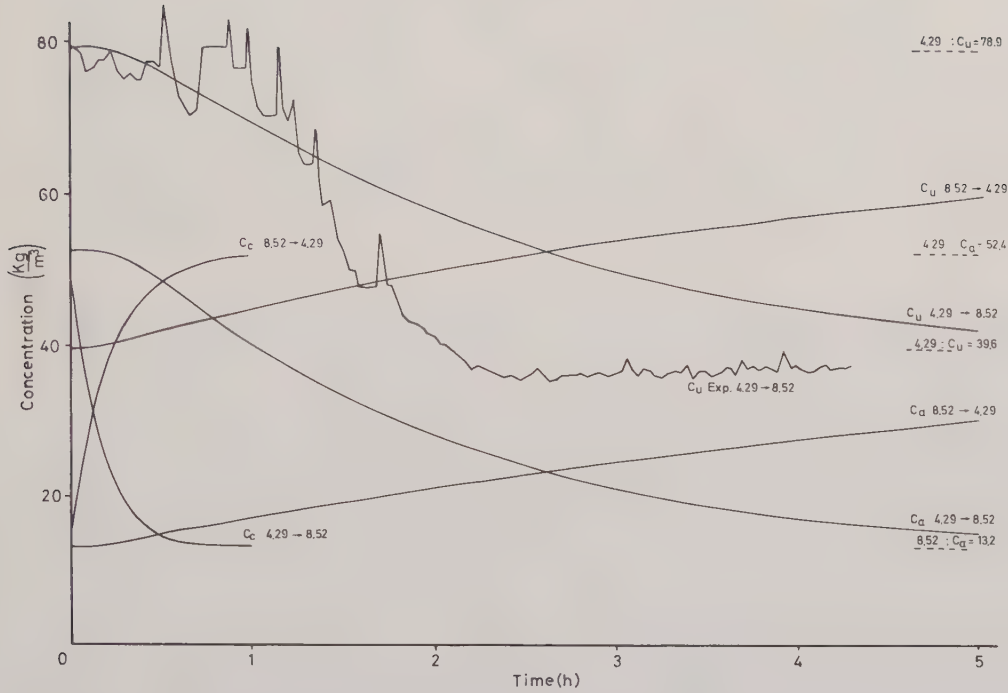


Figure 11. Simulated step responses of c_u , c_c and c_a to solids load change from 8.52 to 4.29 and 4.29 to 8.52 and the experimental step response of c_u to the solids load change from 4.29 to 8.52 $\text{kg/m}^2 \text{ h}$.

However, the simulated responses show a better agreement with experimental data than the responses of the previously discussed models for which Eq. (29) is satisfied.

Euler's method was used to perform the numerical integration in all models above.

STEADY-STATE CONTINUOUS THICKENING

The capacity of a continuous thickener working with an ideal slurry, is generally assumed to be restricted by a critical solids flux. This critical solids flux represents the minimum of the total flux curve, which is shown in Fig. 1. Also with non-ideal slurries, the concept of critical conditions has applicability, if there exists a capacity-restricting critical zone in the thickener. With such a zone, the capacity is not limited by the available compression forces in the thickener.

By making batch settling tests, it is possible to obtain the solids flux curve by settling and to calculate the critical conditions in a continuous thickener with a known underflow rate. Consequently, the theory of capacity-restricting critical conditions is possible to test by comparing continuous thickening data with critical conditions calculated from batch tests.

This has been done by Yoshioka [1], Hassett [3], and Javaheri [9], who found that their experimental results confirmed the theory. However, Hassett and Javaheri pointed out that there were shortcomings in the method. In particular, it was difficult to obtain a well defined minimum on the total solids flux curve. Instead, a flat solids flux band appeared around the minimum. Another deficiency in the method may derive from the fact that the settling velocity is dependent on the hydrodynamic conditions in the slurry [10]. Thus, the settling velocity under quiescent batch conditions can be expected to be higher than in a continuous thickener, at equal concentrations, where vibrations may arise from e.g. the underflow pump.

The experimental work reported in Part II, was performed to test the theory of capacity-restricting critical conditions. A different approach to the test than that used by the authors mentioned above was chosen. Assuming that the theory is correct, there shall exist for a continuous thickener at optimal load a well defined relationship between solids load and underflow concentration. Under these conditions, the critical concentration and the critical solids flux by settling can be obtained from a known solids load/underflow concentration relationship by analytically derived expressions. These expres-

sions were initially derived to model the dynamic behaviour, and are mentioned on page 11. The solids flux curve by settling obtained in this way, can then be compared with experimentally found solids flux data in the constant concentration zone below the dilute zone in the thickener. If this theory is correct, the two flux curves shall be equal. Also, it is possible to test the theory by comparing the thickening results in an optimally loaded thickener at two different load conditions with equal solids loads but different feed flow concentrations. According to this theory, the underflow concentration shall be a function only of the solids load.

The experimental set up used is shown in Fig. 12.

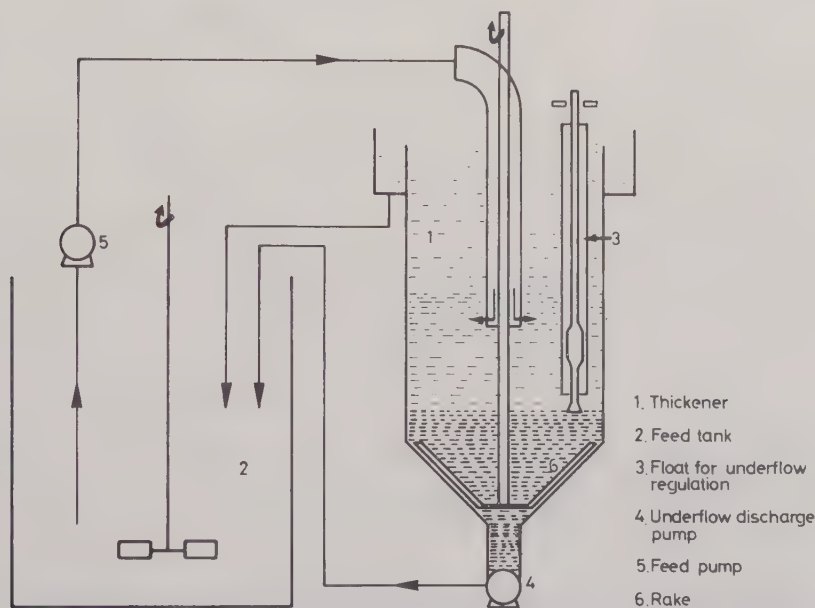


Figure 12. Experimental set up used at steady-state thickening.

The thickener used was the same as the one used on the work with the dynamic behaviour, with the addition of a rake installed in the cone. Besides, sampling-holes were arranged at the thickener wall to obtain the concentration profile. All operating parameters were determined by manual sampling. Solids concentrations were determined gravimetrically by evaporating known slurry volumes. A slurry consisting of calcium carbonate slurried with tap water was chosen for the experiment.

Two feed flow concentrations were used, 23.5 and 54 kg/m³. Some runs were made without raking.

The essential results obtained are shown in Fig. 13.

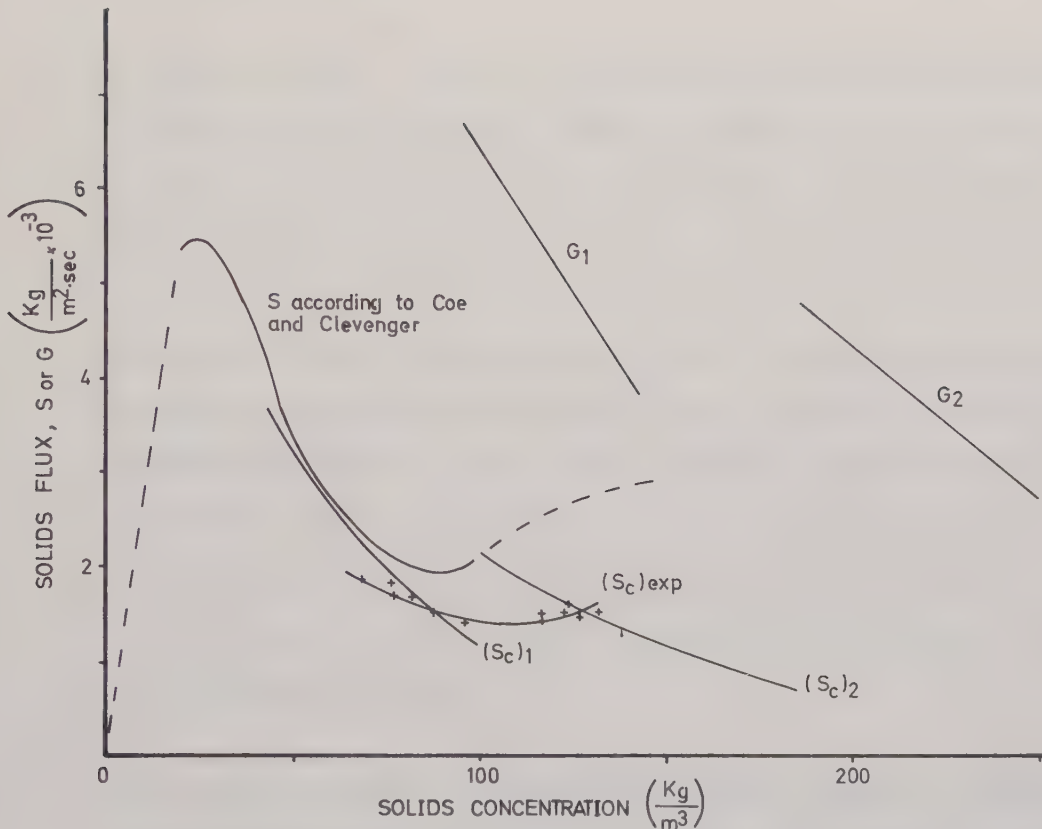


Figure 13. Solids flux curves and the solids load/underflow concentration curves G_1 and G_2 representing optimal loads at feed concentrations 23.5 and 54 kg/m³ respectively.

Six curves are drawn in the figure. G_1 and G_2 represent the underflow concentration/solids load relationships at optimal load and the feed concentrations 54 and 23.5 kg/m³ respectively. These two straight lines are based on experimental conditions with a constant concentration zone, the critical zone, existing below the dilute zone. Thus,

compression forces were not a capacity-restricting factor. Assuming that the theory of a well defined critical solids flux by settling is correct, $(S_c)_1$ and $(S_c)_2$ are derived from G_1 and G_2 respectively. The experimentally found solids flux by settling/concentration data in the critical zone are plotted and through the points the $(S_c)_{exp}$ -curve is drawn. Finally, the S-curve, according to the Coe and Clevenger method, obtained from batch tests with different initial concentrations, is drawn.

The results clearly show that the theory of a well defined relationship between solids load and underflow concentration at optimal load, is not correct for this system. Instead, the feed flow concentration appears as an explicit parameter. As a consequence of the failing theory two S_c -curves appear, one for each feed flow concentration.

Surprisingly enough, the derived S_c -curves, which in fact represent the apparent capacity-restricting fluxes by settling at the prevailing ~~feed~~ concentration give higher S-values than those experimentally found in the critical zone. This is explained by settling conditions above the critical zone which are influenced by the "submerged waterfall" behaviour of the incoming feed. Cross [11] and Scott [12] have observed and discussed the tendency of the feed to behave as a submerged waterfall.

Due to the disturbances of the incoming feed, there will develop a transition zone above the critical zone. In the transition zone non-ideal conditions are assumed to exist with non-uniform solids concentration and slurry underflow rate in each cross-section. The non-uniform solids flux conditions are furthermore supposed to have a positive effect and give a higher total solids flux than that which exists in the critical zone, which is undisturbed by the incoming feed. The comparison between the fluxes is made at an average concentration in a cross-section of the transition zone equal to a uniform concentration in the critical zone.

Regarding the S-curve obtained from batch tests, it is seen that it gives a higher solids flux by settling than the $(S_c)_{exp}$ -curve. Hence, the batch flux is higher than the flux in the critical zone at equal concentrations. As mentioned above, this may be explained by different

kinds of vibrations from the mechanical equipment which decrease the settling velocity in the critical zone if compared to quiescent batch settling.

The influence of the raking action on the thickening capacity was also investigated. It was discovered, that when thickening with a critical zone in the thickener, the capacity was independent of the raking action, whereas with the capacity limited by a compression zone, there was a favourable influence of the raking.

By sampling from the dilute zone, solids flux/concentration data there could be calculated. No significant difference between solids flux data at feed concentration 54 and 23.5 kg/m^3 could be observed. Actually, in the light of the discussed behaviour of the incoming feed, a difference would be expected. Instead, the flux data were shown to be equal to solids flux data obtained from batch settling in a sedimentation balance.

FUTURE DEVELOPMENT

With the results presented in this thesis, a wide field of research appears.

Regarding steady-state thickening, the real capacity-limiting conditions must be investigated. Parameters of interest are the feed flow rate and density and the position and form of the feed well. In order to formulate an accurate theory, these parameters should be coupled with a hydrodynamic model of the thickener. The thickening capacity should then be found as a function of the hydrodynamic conditions and of the solids flux by settling under quiescent conditions. Furthermore, the meaning of the critical flux curve by settling, derived from continuous thickening data with its capacity restricted by a compression zone must be investigated.

Regarding unsteady-state thickening, the found steady-state conditions explain the poor agreement between the simulated models and the experimental data. Thus, it seems clear that a dynamic model can be based on known concentration profiles and underflow concentration/solids load data at steady-states.

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Lund, May 1976

Lars-Göran Eklund

NOTATION

A	thickener area
F	volume feed rate
G	total solids flux = $(L/A) \times c + S$
H1	height of the dilute zone
H2	height of the critical zone
L	volume underflow rate
Q	volume overflow rate
S	solids flux by settling
c	solids concentration, mass solids/volume slurry
c_a	solids concentration in the critical zone at unsteady state, mass solids/volume slurry
Δh	height of uniform concentration layer
t	time
z	vertical coordinate

Subscripts

F	feed flow conditions
Q	overflow conditions
c	critical conditions
cl	clarification zone conditions
d	dilute zone conditions
exp	experimental value
i	i:th uniform concentration layer
u	underflow conditions
z	vertical coordinate

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